NUMERICAL GRAVITY

Ari Maller

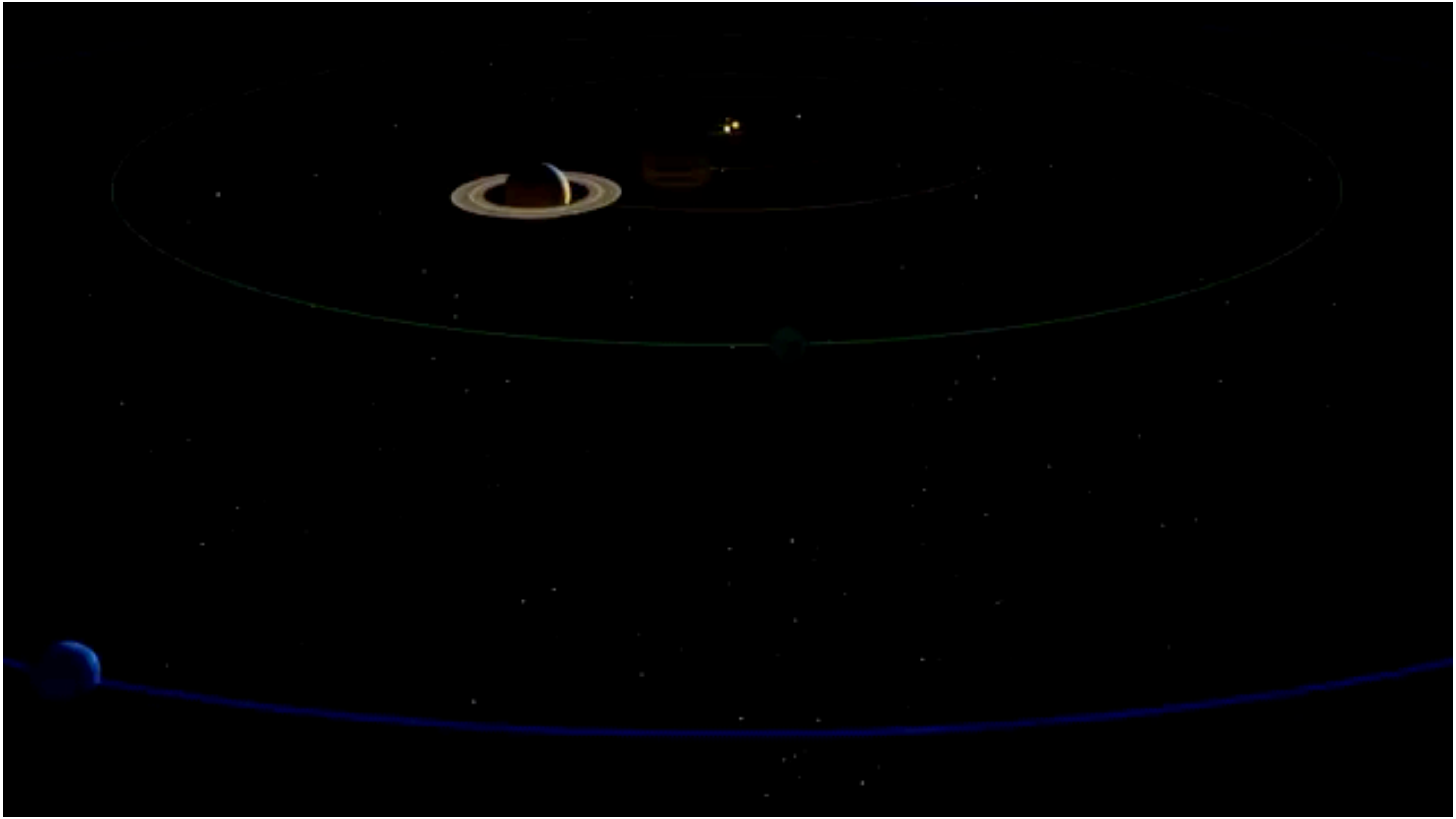
GRAVITY AND ASTROPHYSICS

- The weakest of the 4 fundamental forces only starts to become important at the scale of a moon or planet.
- Therefore problems where gravity is the dominant force are mostly restricted to astrophysics and solar system travel.
- There are 4 regimes
    - Planetary systems
    - Star Clusters
    - Galaxies and Cosmology
    - Merging Black Holes

PLANETARY SYSTEMS

- We can start with planetary systems. Here you have one or more massive objects (stars) and of order 10 much lower mass objects.
- If you have just one star and you ignore the masses of the planets this can be solved exactly (Kepler's Laws).
- However, the masses of the planets slightly perturb the closed orbits. In fact Neptune was discovered by the perturbative effect it had on Uranus.
- These perturbations actually make the solar system unstable and the planets positions can only be modeled for 100Myr with reasonable accuracy.

Orbits of the planets in our solar system.



Note the sizes of the planets are way off so you can see them easily

ADAPTIVE TIME STEPPING

- The first important detail you notice is that the planets orbit on very different time scales. This we know of course from Kepler's Laws, $P^2 = a^3$.
- For Earth to Neptune which orbits at 30 times farther away this means Neptunes period is 164 years.
- This means if you choose a good time step size for Mercury it will be way too small for Neptune, but a good time step for Neptune will give totally unphysical results for Mercury.
- The solution to this is adaptive time-stepping where the step size is different for each planet. So the time step for Earth may be everyday but for Neptune every 128 days.

Gravity Simulation

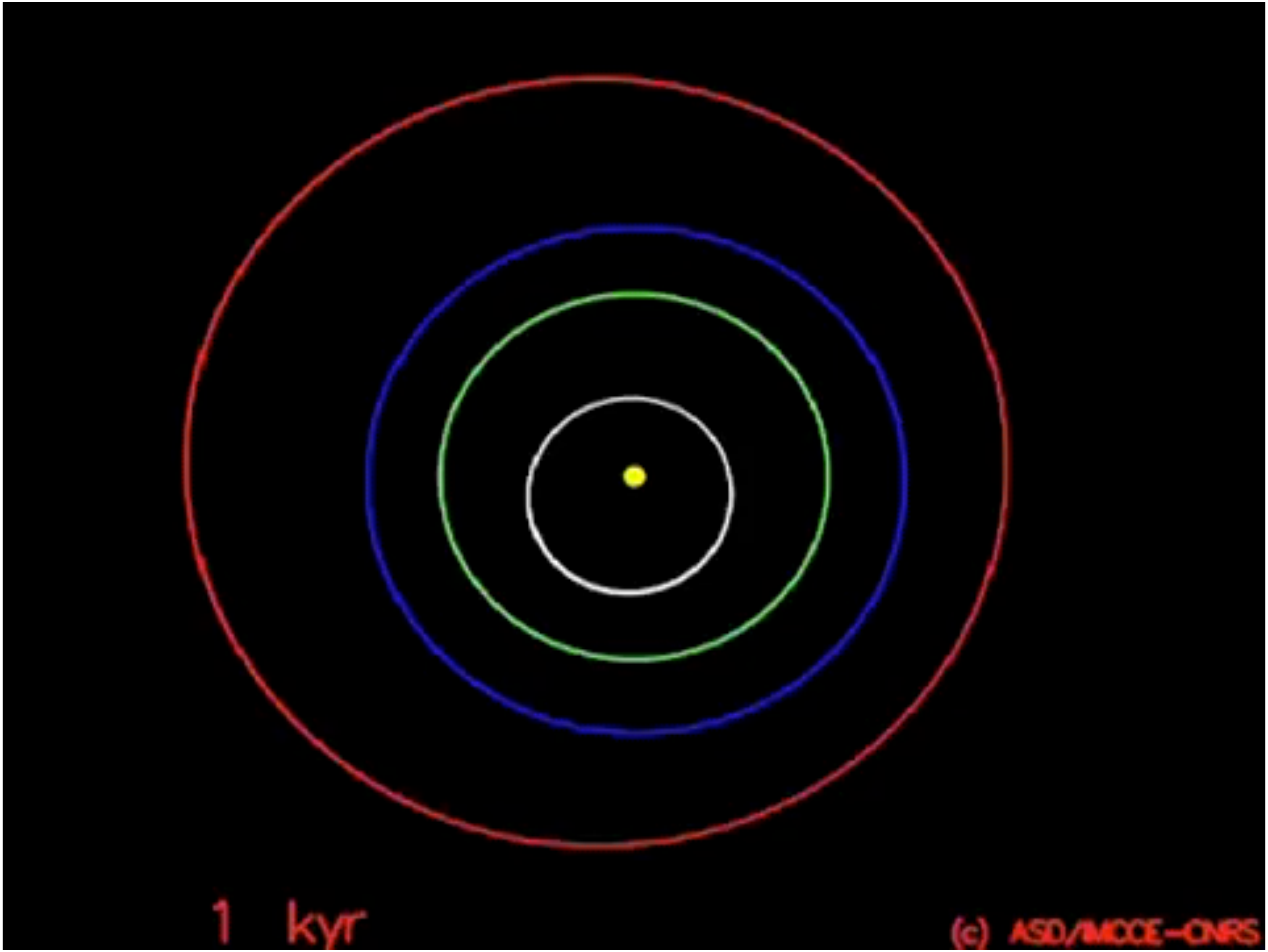
Double Star with a Planet Equals Chaos

Larry Phillips
mathThoughts.com

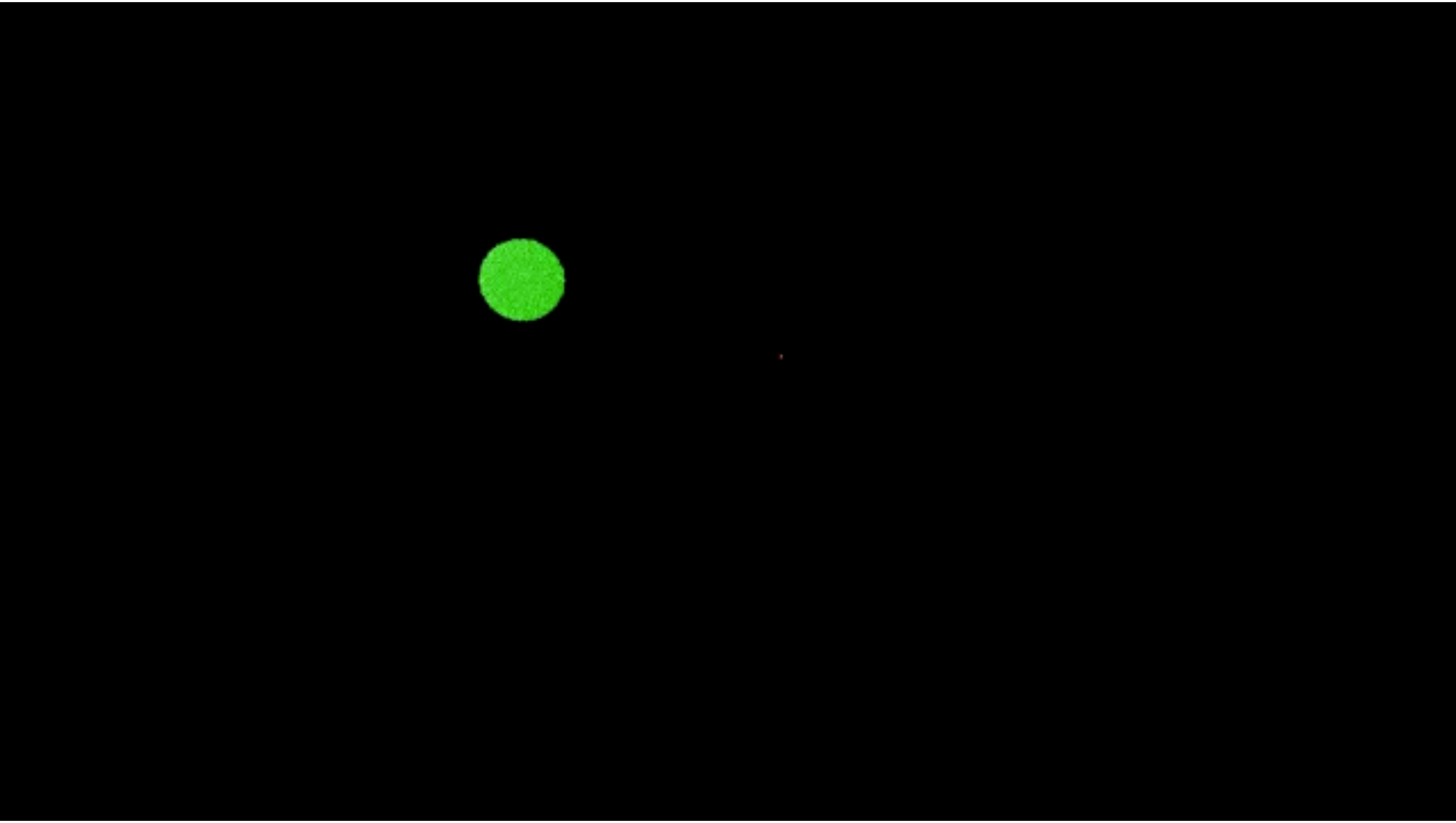
Created using gravity simulation software by
Eugene Butikov
St. Petersburg State University

CHAOS

- Chaos refers to systems where small changes in initial conditions lead to exponentially divergent outcomes.
- Dynamical systems are often chaotic.
- This doesn't mean that they can't be simulated, but that one has to be careful in interpreting the result of the simulation.
- For example one can simulate the evolution of the solar system many times with slightly different initial positions (within the known uncertainties). From this one can learn things about how chaotic the system is, not the actual locations of the planets in the future.
- For example is has been shown that Mercury is so chaotic that a 0.38mm difference in its position today makes predicting its eccentricity in 200Myr impossible.


1 kyr
(c) ASD/IMCCE-CNRS

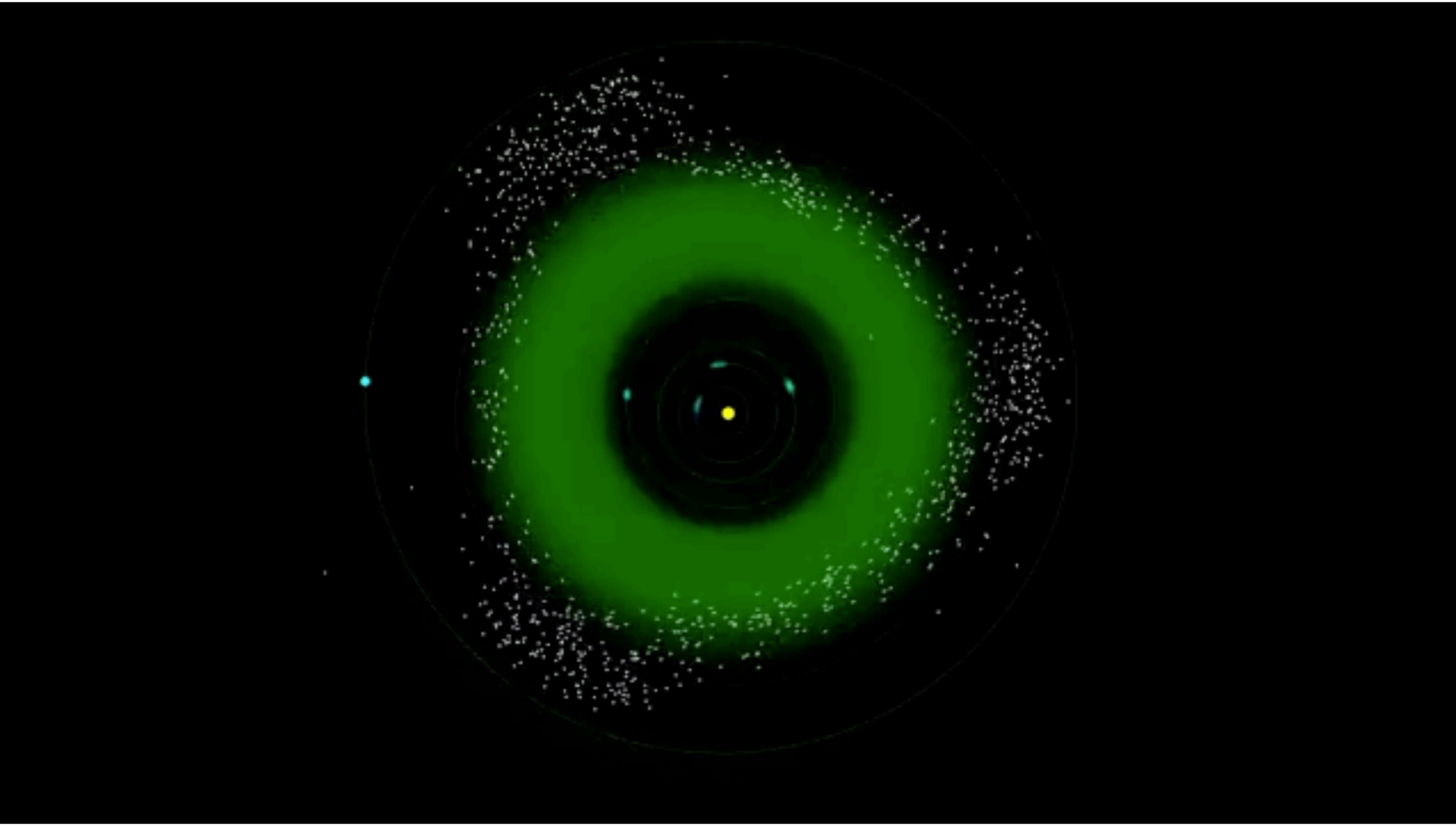
Binary star system, the companion is not shown.



EXTENDED BODIES

- Stars, planets and moons are not point particles, the are extended objects.
- Gravitational interactions effect these bodies, in particular tidal forces which can remove material from a star or destroy a moon.
- Tidal interactions between the Earth and Moon are slowing the Earth’s rotation and most of the solar systems moons are phase locked with their planet.
- To add these features a model of the hydrodynamics / geophysics for the objects must be added.

Simulation of astroids in resonance with Jupiter



Reference frame is rotating with Jupiter

RESONANCE

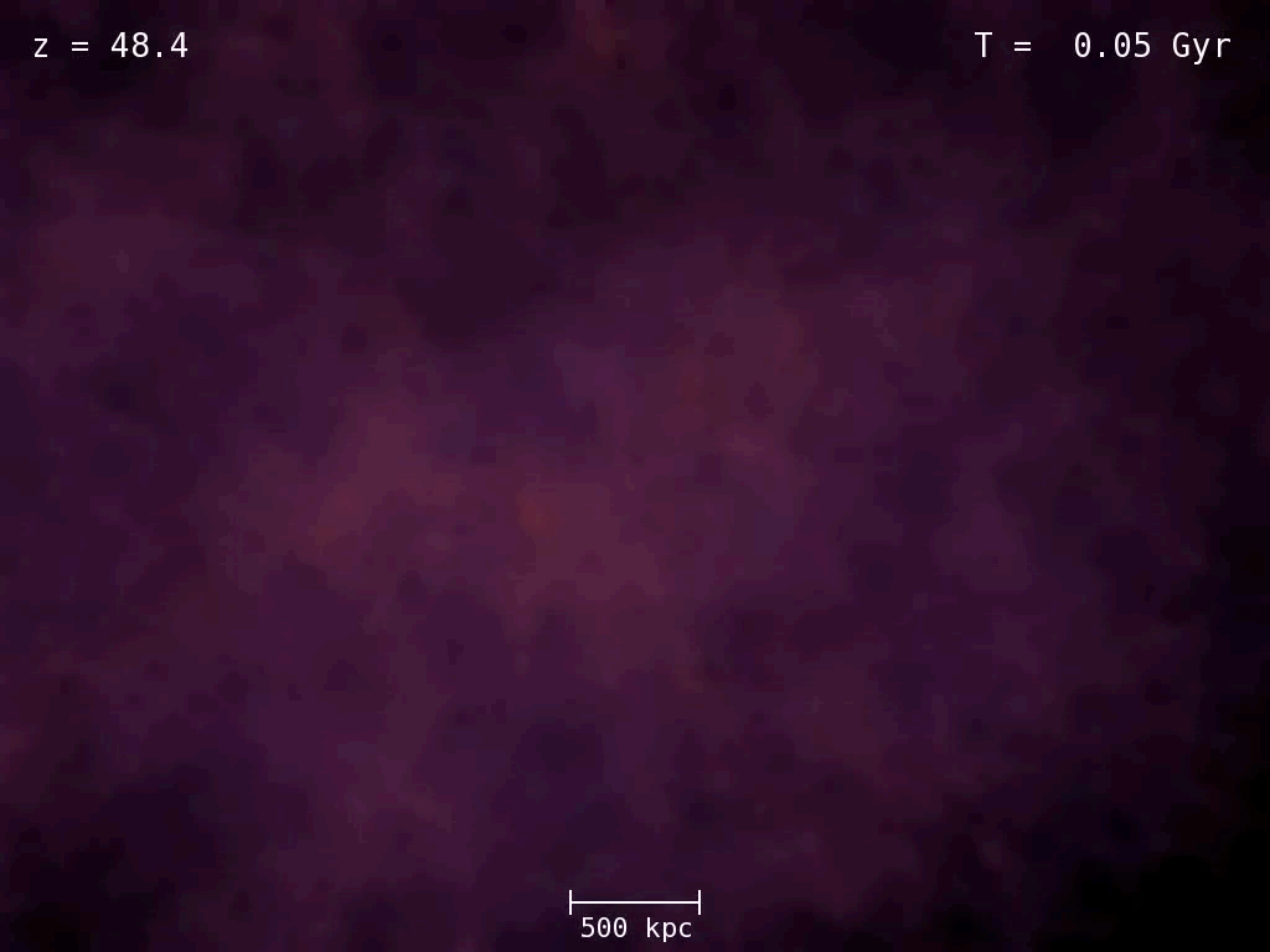
- Another difficult aspect of dynamical simulations is that resonances can often be very important.
- Resonances are challenging to simulate because small errors will ruin the resonance failing to correctly capture the dynamics.
- A resonance will usually occur in a very narrow region of phase-space meaning one has to sample the phase-space densely to capture the resonance and solve the motion accurately to stay in the resonance.
- This can be very challenging and simulations often have a hard time capturing resonances. If resonance are important to the problem it is critical to be aware of that.

COSMOLOGY

- Let us now move to the largest scale, cosmology.
- The universe evolves as tiny perturbations grow over time due to gravity. Since the universe is 80% dark matter we can get an approximate view of its evolution ignoring all other forces completely and treating it as only subject to gravity.
- A representative volume in the universe is of order 100 Mpc^3, or 300 Mly^3 for non astronomers. Simulating this with billions of particles still requires that each particle is millions of solar masses.
- The actual dark matter particle mass is maybe 1GeV so a single particle in the simulation is maybe $\sim 10^{55}$ dark matter particles.

SCALES AS N^2

- The first problem we see is that a direct approach would scale as the number of particles squared. This would limit us to ~100,000 particles maximum which would not resolve any of the structures we would like to see.
- The fundamental realization to get around this problem is that when particles are far away an error in their exact position has little effect in their contribution to the force.
- Thus if we had one hundred particles far away, instead of summing 100 forces, we could take their average location and calculate one average force from them.
- One way to implement this is to build a tree of the particles based on their spatial locations. Then for our force calculation one can use the exact particle positions for nearby particles, but use the average value for farther away nodes.
- Another approach is create a grid and calculate how many particles are in the grid. The for the force calculation use the particles in the same grid, but the average density for all other cells.
- This last approach can be made even more efficient if the grid is adaptively refined so that the cell size is small in dense regions, but large in low density regions. This approach is called adaptive mesh refinement or AMR.


z = 48.4
T = 0.05 Gyr
500 kpc

RELAXATION TIME

- In astrophysics relaxation time is an estimate of the time it would take for a star to have its velocity substantially changed by 2-body interactions.
- It is given by

$$T_r = \frac{v^3}{6G^2 m \rho \ln \Lambda}$$

- where v is the average star's velocity, m is the average star's mass, ρ is the density and ln Λ is the Coulomb logarithm which has value around 15.
- The relaxation time divides gravitational systems into collisional and collisionless systems.
- Most astrophysical systems are collisionless, only dense stellar clusters have relaxation times that can be as short at 100Myr.

GRAVITATIONAL SOFTENING

- If our system is collisionless then 2 body encounters in our simulation are not physical. We actually want to remove them.
- This is done by changing the gravitational force to be $(r+\varepsilon)^{-2}$ where ε is called the smoothing length.
- In this way we never get the very large accelerations that would occur when particles are very close, which in this case is good because are 'particles' are not point masses but instead represent spatially extended distributions of vast numbers of particles.
- However, if the system is collisional then we need to resolve the encounters which are computationally very expensive because we need to use very short time steps.

Even simulations that focus on just one galaxy are collisionless

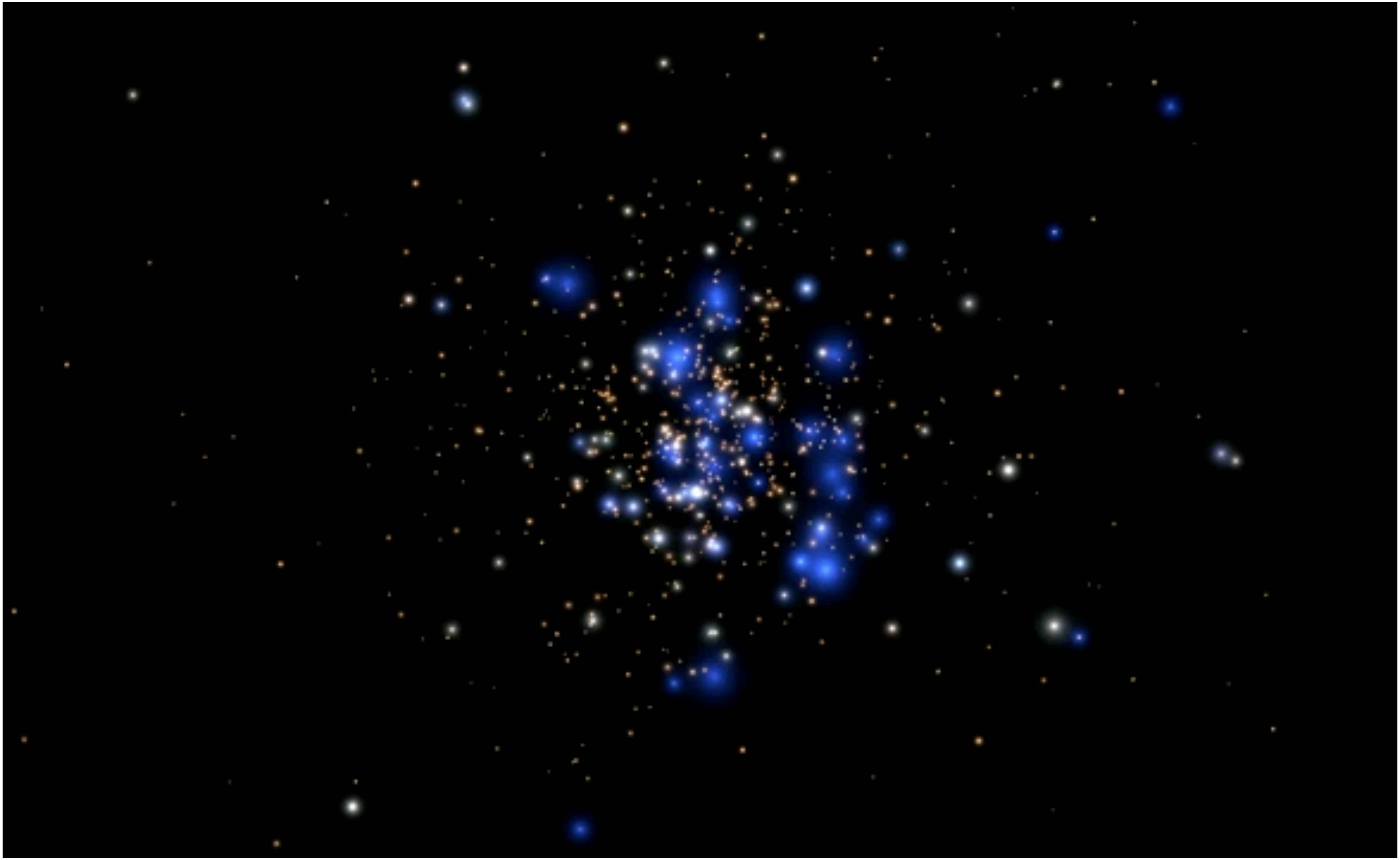
In this case one can use much higher resolution.

STAR CLUSTERS

- Stars are born in clusters of thousands of stars. These open clusters do not last for that long as the stars become unbound.
- There are also globular clusters that contain 10^5 - 10^6 stars. Their origins are unclear, but many are as old as the universe.
- In both cases the relaxation times are fairly short and these systems can not be modeled with gravitational softening.
- They can thus be very challenging to simulate as you would need to have one particle per star.
- Other complications are that the stars themselves evolve, shedding mass and eventually exploding as supernova.

Dynamical evolution of a cluster using Nbody6. Sizes are proportional to sizes of the stars and stars disappear because they supernova.

Evolution of a young star cluster, the cluster dissolves over time.



BLACK HOLE MERGERS

- As the possibility of a gravitational wave detector become a reality, simulations of black hole - black hole mergers became a priority.
- These simulations are actually easier than you might think, as black holes are the simplest possible objects, only having mass and spin.
- In addition we are only interested in the gravitational waves emitted by these mergers which propagate to infinity and thus are immune to errors that fall off with distance from the merger.
- Mergers of black holes and neutron stars are much more difficult because one needs a model of the structure of the neutron star.

In this black hole - black hole merger simulation the colors are showing the gravitational waves emitted by the merger. The white line at the bottom shows gravitational wave signal created by the merging black holes.